There are two basic theories for verification of the stability of an assumed arch section. The *elastic theory* considers the arch as a curved beam subject to moment and shear, whose stability depends on internal stresses. For arches subject to non-symmetrical loading that can cause tensile stress development, the elastic theory provides the most accurate method of analysis. There are many methods of elastic analysis for arch design, but in most instances their application is complicated and time consuming. Such detailed engineering discussions are beyond the scope of this book, and the reader is referred to Valerian Leontovich's *Frames and Arches* (McGraw-Hill, New York, 1959) for further information.

A second theory of analysis is the *line-of-thrust method,* which considers the stability of the arch ring to be dependent on friction and the reactions between the several arch sections or voussoirs. In general, the line-of-thrust method is most applicable to symmetrical arches loaded uniformly over the entire span or subject to symmetrically placed concentrated loads. For such arches, the line of resistance (which is the line connecting the points of application of the resultant forces transmitted to each voussoir) is required to fall within the middle third of the arch section, so that neither the intrados nor extrados of the arch will be in tension (*see Fig. 11-15* for arch terminology).

11.2.2 Graphic Analysis

The simplest and most widely used line-of-thrust method is based on the hypothesis of "least crown thrust," which assumes that the true line of resistance of an arch is that for which the thrust at the crown is the least possible consistent with equilibrium. This principle can be applied by static methods if the external forces acting on the arch are known and the point of application and direction of crown thrust are assumed. Normally, the direction of the crown thrust is assumed as horizontal and its point of application as the upper extremity of the middle one-third of the section (i.e., two-thirds the arch depth from the intrados). This assumption has been proven reasonable for symmetrical arches loaded symmetrically, but is not applicable to non-symmetrical or partially distributed uniform loads.

With these assumptions, the forces acting on each section of an arch may be determined by analytical or graphic methods. The first step in the procedure is to determine the joint of rupture. This is the joint for which the tendency of the arch to open at the extrados is the greatest and which therefore requires the greatest crown thrust applied to prevent the joint from opening. At this joint, the line of resistance of the arch will fall on the lower extremity of the middle third of the section. For minor segmental arches, the joint of rupture is ordinarily assumed to be the skewback of the arch. (For major arches with higher rise/span ratios, this will not be true.) Based on the joint of rupture at the skewback and the hypothesis of least crown thrust, the magnitude and direction of the reaction at the skewback may be determined graphically (*see Fig. 11-16*).

In this analysis, only one-half of the arch is considered, since it is symmetrical and uniformly loaded over the entire span. *Figure 11-16A* shows the external forces acting on the arch section. For equilibrium, the lines of action of these three forces ($W/2$, H, and R) must intersect at one point as shown in *Fig. 11-16B*. Since the crown thrust (H) is assumed to act horizontally, this determines the direction of the resisting force (R). The magnitude of the resistance may be determined by constructing a force diagram as indicated in *Fig. 11-16D*. The arch is divided into voussoirs and the uniform load trans-

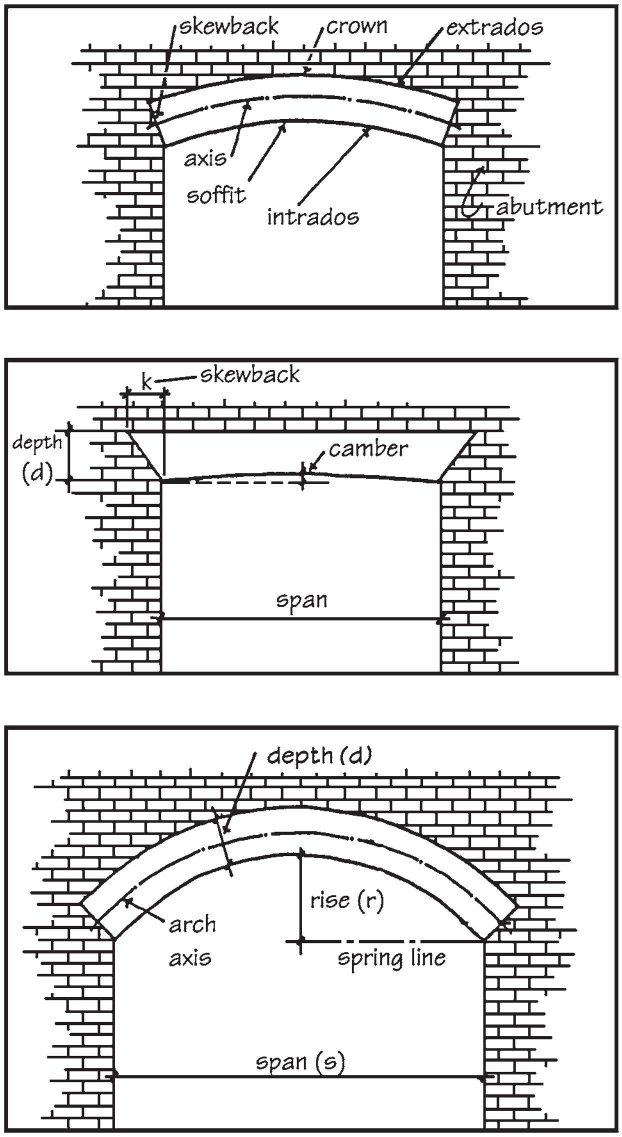

Figure 11-15 Arch terminology (see the Glossary in Appendix A). (*From BIA* Technical Note 31A.)

formed into equivalent concentrated loads acting on each section (*see Fig. 11-16C*). Starting at any convenient point (in this example, between the reaction and the first load segment past the skewback), numbers are placed between each pair of forces, so that each force can subsequently be identified by a number (i.e., 1–2, 5–6, 7–1, and so on). The side of the force diagram which represents $W/2$ (*Fig. 11-16D*) is divided into the same number of equivalent loads, and the same numbers previously used for identification are placed as shown in *Fig. 11-16E* to identify the forces in the new force diagram. Thus, the line 7–1 is the skewback reaction, 6–7 the horizontal thrust, and so on. From the intersection of H and R (7–1 and 6–7) a line is drawn to each intermediate point on the leg representing $W/2$.

The equilibrium polygon may now be drawn. First extend the line of reaction until it intersects the line of action of 1–2 (*see Fig. 11-16F*). Through